Information Geometrical Framework for Analyzing Belief Propagation Decoder

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Abstract

The mystery of belief propagation (BP) decoder, especially of the turbo decoding, is studied from information geometrical viewpoint. The loopy belief network (BN) of turbo codes makes it difficult to obtain the true “belief” by BP, and the characteristics of the algorithm and its equilibrium are not clearly understood. Our study gives an intuitive understanding of the mechanism, and a new framework for the analysis. Based on the framework, we reveal basic properties of the turbo decoding.

1 Introduction

Since the proposal of turbo codes[2], they have been attracting a lot of interests because of their high performance of error correction. Although the thorough experimental results strongly support the potential of this iterative decoding method, the mathematical background is not sufficiently understood. McEliece et al.[5] have shown its relation to the Pearl’s BP, but the BN for the turbo decoding is loopy, and the BP solution gives only an approximation.

The problem of the turbo decoding is a specific example of a general problem of marginalizing an exponential family distribution. The distribution includes higher order correlations, and its direct marginalization is intractable. But the partial model with a part of the correlations, can be marginalized with BP algorithm exactly, since it does not have any loop. By collecting and exchanging the BP results of the partial models, the true “belief” is approximated. This structure is common among various iterative methods, such as Gallager codes, Bethé approximation in statistical physics[4], and BP for loopy BN.

We investigate the problem from information geometrical viewpoint[1]. It gives a new framework for analyzing these iterative methods, and shows an intuitive understanding of them. Also it reveals a lot of basic properties, such as characteristics of the equilibrium, the condition of stability, the cost function related to the decoder, and the decoding error. In this